

# On the constituent counting rules for hard exclusive processes involving multiquark states

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At high energies, the cross section of a hard exclusive process at finite scattering angle falls off as a negative power of the center-of-mass energy  $\sqrt{s}$ . If all involved quark-gluon compositions undergo hard momentum transfers, the scaling of the fall-off is determined by the underlying valence structures of the initial and final states, known as the constituent counting rules. It was argued in the literature that the counting rules are a powerful tool to determine the valence degrees of freedom inside multiquark states when applied to exclusive production processes. However, we demonstrate that for hadrons with hidden flavors the naive application of the constituent counting rules is problematic, since it is not mandatory for all components to participate the hard scattering at the scale  $\sqrt{s}$ . The correct scaling rules can be obtained easily by using effective field theory. A few examples involving the  $Z_c(3900)^\pm$  and  $X(3872)$  are discussed.

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The concept of valance quarks has played a major role in the classification of the hadronic states. Hundreds of hadrons were discovered in experiments, and most of them are accommodated in the quark model: mesons and baryons are composed of a quark–antiquark pair and three quarks, respectively. Here, the terminology quark refers to the valence degrees of freedom. Hadrons beyond such configurations are dubbed as exotic, and searching for them, in particular those with exotic quantum numbers which cannot be formed by the above mentioned simple configurations, is of utmost importance in understanding the low-energy nonperturbative quantum chromodynamics (QCD) because color confinement allows such color-singlet states.<sup>1</sup> Thanks to worldwide experiments during the last decade at  $e^+e^-$  and hadron colliders, a plethora of new structures as candidates of various hadron resonances were reported with properties different from quark model expectations, and it is probable that some of them could be interpreted as exotic multi-quark states. Most of these new discoveries are in the heavy quarkonium mass region (for reviews, see, e.g., Refs. [5–8]). Among them, a milestone was the discovery of the  $X(3872)$  by the Belle Collaboration in 2003 and confirmed by several other experiments later [9–12]. Since then one key topic in hadron physics is the study of these observed structures.

Taking the  $XYZ$  states in the charmonium mass region as an example, a number of interpretations have been proposed including normal quarkonia, hybrid states, compact multiquark states, hadro-charmonia, hadron molecules and effects due to kinematical singularities. Most of these interpretations are based on quark model notations assuming explicitly or implicitly that the number of (valence) quarks is well defined, even when discussing the production of multiquarks at very high momentum transfer. Then, a central question discussing the observed candidates of exotic hadrons (including proposing new measurements of the properties of these states and searching for new structures) is: how can one model-independently determine the valence quark-gluon composition of a hadron? In a special case of a hadron located very close to an  $S$ -wave threshold of two other hadrons, one can in fact measure the valence hadron component since hadrons being asymptotic states can go on shell. However, it becomes very complicated when one tries to determine the quark-gluon components. This is because of confinement which tells us that quarks and gluons are not asymptotic states and can not be measured directly in experiments. Let us define the numbers of quarks and antiquarks as  $n_q$  and  $n_{\bar{q}}$ , respectively. In a system with hidden flavor,  $n_q - n_{\bar{q}}$  is well defined because of baryon number conservation, while  $n_q + n_{\bar{q}}$  is not. In particular, one does

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<sup>1</sup> Classical large  $N_c$  arguments state that  $q\bar{q}q\bar{q}$  tetraquark states are absent in the large  $N_c$  limit (see Refs. [1, 2]). However, this conclusion was challenged in Ref. [3] where it was argued that tetraquark states can exist in the large  $N_c$  limit with narrow widths. This was elaborated on in Ref. [4].

not expect the latter to take a definite value for a given hadron in processes happening at different energy scales. However, the latter is the key quantity discussed in some papers in the literature, and we conclude that the conclusions of these papers are thus model-dependent and sometimes problematic. This argument can be made more clear by showing why the constituent “counting rules” fail for multi-quark states in hard exclusive processes.

Recently, it has been argued in Refs. [13–17] that the differential cross section for high energy production of multi-quark states should scale with a certain power of  $s$ , the center-of-mass energy squared, predicted based on their expected valence quark structures. More explicitly, for a generic process  $a + b \rightarrow c + d$ , the cross-section is argued to obey the behavior [13–17]:

$$\frac{d\sigma}{dt} \sim s^{2-n} f(\theta_{cm}), \quad (1)$$

with  $n = n_a + n_b + n_c + n_d$ . Here,  $s$  and  $t$  are the conventional Mandelstam variables,  $\theta_{cm}$  is the scattering angle in the center-of-mass frame, and  $n_h$  is the number of constituents in the particle  $h$ . Here  $a, b, c, d$  denote generic leptons, photons or hadrons including multi-quarks. A fundamental particle like a quark, electron, or a photon has  $n_i = 1$ . An ordinary meson has  $n_i = 2$ , a meson-meson molecule or a compact tetraquark has  $n_i = 4$ , and a pentaquark has  $n_i = 5$ , all of which amount to  $n_q + n_{\bar{q}}$  defined before. The investigated processes include  $\pi^- + p \rightarrow K^0 + \Lambda(1405)$ ,  $\gamma + p \rightarrow K^+ + \Lambda(1405)$  [13, 14, 17], exclusive electron-positron annihilation [15, 16] and so on. Moreover, within the tetraquark framework, the authors of Ref. [16] have argued that based on distinctive fall-offs in  $s$  of the cross sections, it is possible to distinguish whether the tetraquarks are segregated into di-meson molecules, diquark-antidiquark pairs, or more democratically arranged four-quark states.

Were the constituent counting rules correct, it would provide a very powerful and straightforward tool to access the valence quark structures of the exotic hadrons. But unfortunately as we have argued above and will show in more detail below, for hadrons with hidden-flavor quarks it is problematic to apply such a naive constituent counting rule. To be explicit, we will first consider a simpler example involving only ordinary mesons,  $e^+e^- \rightarrow VP$ , with  $V$  and  $P$  denoting ordinary light flavor vector and pseudoscalar mesons, respectively. This reaction does not follow the naive scaling rule shown in Eq. (1). Then we will adopt the framework of effective field theory and point out the problems in the derivation of the misleading scaling behavior in Eq. (1).

At very high energy with  $\sqrt{s} \gg \Lambda_{\text{QCD}}$ , exclusive processes can be understood in QCD perturbation theory [18]. The scaling behavior exists in the factorization limit and can be formally derived by matching the full theory, QCD, to an effective field theory. When factorization is applicable,

one can formally separate the interactions according to the involved scales:

$$T\exp\left[i\int d^4x\mathcal{L}_{\text{int}}(x)\right] = T\exp\left[i\int d^4x\mathcal{L}_{\text{int}}(x)\right]_{>\mu} \times T\exp\left[i\int d^4x\mathcal{L}_{\text{int}}(x)\right]_{<\mu} \quad (2)$$

where  $T$  stands for time ordering,  $\mu$  is the factorization scale and for high-energy processes we should use  $\mu \sim \sqrt{s}$  in order to suppress the large logarithms in higher order contributions. Perturbation theory at high energies allows one to express the matrix elements of Heisenberg operators in terms of free local operators and the interaction terms. By including the interaction, we have:

$$\begin{aligned} \langle f|\mathcal{O}_H(0)|i\rangle &= \langle f|T\left[\mathcal{O} \times \exp[i\int d^4x\mathcal{L}_{\text{int}}(x)]_{>\mu} \times T\exp[i\int d^4x\mathcal{L}_{\text{int}}(x)]_{<\mu}\right]|i\rangle \\ &\sim \langle f|T\left[\mathcal{O}' \times \exp[i\int d^4x\mathcal{L}_{\text{int}}(x)]_{<\mu}\right]|i\rangle \\ &\equiv \langle f|\mathcal{O}'_{H,\mu}|i\rangle, \end{aligned} \quad (3)$$

where in the last step we have formally integrated out the interactions above the factorization scale  $\mu$  using the operator product expansion, and obtained a new set of generic low-energy effective operators  $\mathcal{O}'$ . The interaction below  $\mu$  contains no information on the  $1/\sqrt{s}$  scaling and thus the scaling behavior can be obtained by counting the pertinent number of constituents in the operator  $\mathcal{O}'$ . It is possible to include the effects due to the renormalization group and resummation of double logarithms known as Sudakov logarithms. For simplicity, we do not consider these effects here since the leading power behavior will be unaltered. An implication of the above analysis is that the  $s$ -scaling of a process at high energies is given by that contained in the operator  $\mathcal{O}'$ , and one gets the scaling easily by identifying the number of lines attached to the vertex described by that operator, which is depicted as a circled cross in the figures in the examples to be discussed below, with the value of  $n$  in Eq. (1).

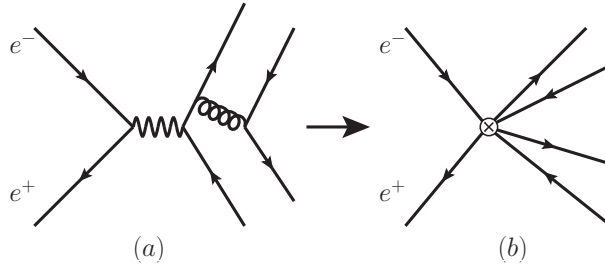


FIG. 1: Feynman diagrams for the  $e^+e^- \rightarrow VP$  with the quark-antiquark pair produced by a gluon. (a) and (b) stand for the diagram in the full theory and the EFT, where the hard propagators are shrunk to a point depicted as a circled cross, respectively.

It is straightforward to apply the constituent counting rules to the  $e^+e^-$  annihilation into two light mesons whose typical Feynman diagram is given in Fig. 1 (a). The  $s$  power dependence is normally determined by the constituent counting rule in Eq. (1) [18] with  $n = 6$ . For the  $e^+e^- \rightarrow VP$  with  $V$  and  $P$  being a vector and pseudoscalar meson, respectively, the differential cross section scales as

$$\frac{d\sigma(e^+e^- \rightarrow VP)}{dt} \propto \frac{1}{s^5}, \quad (4)$$

which differs from Eq. (1) that would give  $1/s^4$  because of an additional suppression factor of  $1/s$  due to helicity flip. Recent measurements of the process  $e^+e^- \rightarrow KK^*$  by the Belle Collaboration [19] at 10.58 GeV and CLEO Collaboration [20] are consistent with the above scaling in Eq. (4) (see also results from BES [21] and BaBar [22]).

However, if the vector meson is composed of a pair of quark and antiquark with the same flavor, like the  $\rho^0, \omega, \phi$  and  $J/\psi$  mesons, the scaling behavior will be different at high energies. We show a production mechanism in Fig. 2 (a), which leads to the scaling behavior:

$$\frac{d\sigma(e^+e^- \rightarrow VP)}{dt} \propto \frac{1}{s^3}, \quad (5)$$

as can be read off from Fig. 2 (b) where the effective interaction vertex has  $n = 5$ . It is necessary to point out that this production mechanism is suppressed by the fine structure constant  $\alpha_{\text{em}} \sim 1/137$  and thus less important at low energies. But apparently at very high energies this new diagram will provide the dominant contribution and it gives a scaling rule different from the one by naively counting the number of valence quarks in the mesons [23].

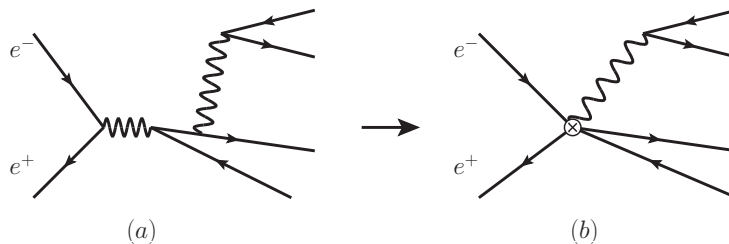


FIG. 2: Feynman diagrams for the process  $e^+e^- \rightarrow VP$  with both  $V$  and  $P$  being neutral. The neutral vector meson is produced via a photon. Integrating out high-off-shell propagators, one obtains (b) from (a).

It is straightforward to understand the above behaviors through the diagrams in Figs. 1 and 2. In Fig. 1 (a), all internal propagators have typically large off-shellness:  $p^2 \sim s$ . In Fig. 2 (a), the virtuality of the second photon, equal to the mass square of the vector meson, is much smaller.

Thus, to accommodate with the constituent scaling rule, one can technically count the valence degrees of freedom of the neutral vector meson as  $n_i = 1$  since it is produced by a photon, which amounts to count the number of lines attached to the effective vertex. The lesson one can learn from the above example is: not all ingredients undergo the hard momentum transfer at the scale  $\sqrt{s}$ . The scaling of the fall-off is determined by the leading-power operator at the scale  $\mu = \sqrt{s}$  which has a nonzero matrix element with the hadron. Actually, the original constituent counting rule is applicable at finite scattering angles. If the scattering angle is small, at least two of the involved particles are collinear which will also spoil the constituent counting rule.

Let us switch to the exclusive production of multi-quark states, and take the reaction  $e^+e^- \rightarrow Z_c^\pm \pi^\mp$  as an example. In Ref. [16], it has been argued that its cross section in the  $s \rightarrow \infty$  limit obeys the scaling

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}cd\bar{u})\pi^-(\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \stackrel{?}{\propto} \frac{1}{s^4}, \quad (6)$$

where the  $Z_c^+(\bar{c}cd\bar{u})$  is a tetraquark state composed of two quarks and two antiquarks. We have put a question mark to the above scaling behavior since we believe that it is problematic at very high energies. We show a production mechanism in Fig. 3 (a). In this diagram, the heavy quark pair  $\bar{c}c$  is generated from the QCD vacuum, and thus such a contribution is suppressed by  $\mathcal{O}(1/m_c^2)$ . But since the main focus of this work is the scaling behavior in terms of the collision energy, we are less interested in the  $1/m_c^2$  suppression. Integrating out the off-shell intermediate propagators at the scale  $\sqrt{s}$  we find that the  $Z_c$  behaves as an ordinary  $\bar{q}q$  meson and the  $s$  dependence scaling of the cross-section is determined by the light quarks of the  $Z_c$ :

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+\pi^-)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \propto \frac{1}{s^2}, \quad (7)$$

which can again be obtained by counting the number of lines attached to the effective vertex. Apparently, this production mechanism will become dominant at very high energies with  $\sqrt{s} \gg m_c$ .

A further example is the reaction  $e^+e^- \rightarrow Z_c^\pm Z_c^\mp$  argued to exhibit the fall-off scaling [16],

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}cd\bar{u})Z_c^-(\bar{c}c\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \stackrel{?}{\propto} \frac{1}{s^6}, \quad (8)$$

which should be corrected as

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}cd\bar{u})Z_c^-(\bar{c}c\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \propto \frac{1}{s^2}. \quad (9)$$

The above results are applicable for the longitudinal polarization, while the transverse polarized case should be further suppressed by  $1/s$ .

It is noteworthy to mention that the above discussions on the  $Z_c^\pm$  production are valid in both the tetraquark (diquark-anti-diquark or a democratically arranged four-quark state), and hadronic molecular pictures. The scaling behavior is the same in both scenarios, and therefore one can hardly make a statement distinguishing a compact tetraquark from a meson-meson molecule using hard exclusive processes.

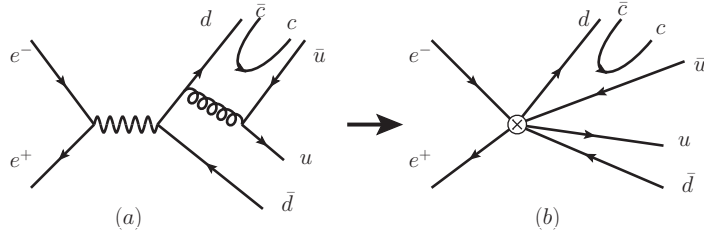


FIG. 3: Feynman diagrams at leading power in  $1/s$  for the reaction  $e^+e^- \rightarrow Z_c^-(\bar{c}c\bar{u}d)\pi^+$  at very high energies. The  $\bar{c}c$  quark pair is generated by QCD interactions at the scale  $\mu \sim m_c$  which is much lower than  $\sqrt{s}$ .

Ref. [17] has applied the naive counting rule to the photoproduction of hyperon resonances and attempted to study the  $\Lambda(1405)$ . The  $\Lambda(1405)$  is expected to be a  $\bar{K}N-\pi\Sigma$  bound state [24–27]. The fitted constituent number is energy-dependent, which can be understood since the constituent counting rule determines the asymptotic behavior in the large energy limit, and will be distorted by finite energy corrections. At the largest collision energy, however the obtained constituent number is consistent with three for the  $\Lambda(1405)$ , despite of the large errors. The fact that  $n = 3$  does not imply that the  $\Lambda(1405)$  is an ordinary  $uds$  baryon but instead it shows that the production mechanism of the  $\Lambda(1405)$  at short distances involves three quarks.

Regarding the notable exotics candidate  $X(3872)$ , an important task in understanding its nature involves the discrimination of a quark-antiquark configuration, a compact multiquark configuration and a hadronic molecule (we refer to the recent review [7] which summarizes nicely the literature). Unlike the  $Z_c^\pm$ , the  $X(3872)$  is neutral, and both the light quark-antiquark pair and charm-anticharm quark pair are hidden. So in hard exclusive processes, the  $X(3872)$  can be produced at short distances by two sets of operators:

$$\langle X|\bar{c}\Gamma c|0\rangle, \quad \langle X|\bar{q}\Gamma q|0\rangle, \quad (10)$$

with  $q$  being a light  $u/d$  quark field, and  $\Gamma$  denoting the Lorentz structure of the operator to produce the  $X$ . The explicit form of the contributing operators depends on the process. For

instance, Ref. [28] has explored the inclusive production of  $X(3872)$  in  $B$  decays and at hadron colliders, and pointed out that the most important term in the factorization formula should be the color-octet  $^3S_1$  term. In exclusive  $B_c$  decays into  $X(3872)$ , the  $\langle X|\bar{c}\Gamma c|0\rangle$  contributes [29], and ratios of branching fractions can be predicted with a high precision under this mechanism, no matter whether the long-distance nature of  $X$  is given by a tetraquark or hadronic molecule composition.

To understand the internal structure of exotic hadron candidates, it is essential to figure out the corresponding valence quark-gluon compositions. However, at low energies, since the effective degrees of freedom are hadrons, and only integrated quantities can be observed, it is very hard to determine the valence components. One might hope that cross sections of the exclusive productions of these hadrons may be used to determine the valence components of a multiquark state since there are constituent counting rules for hard exclusive processes determined by the number of elementary particles involved. However, we have argued in this paper that multiquark states with hidden flavors do not have to follow the scaling rule by simply counting the number of valence quarks and antiquarks. The reason is that the hidden-flavor pair could be produced by a much softer momentum exchange. In the spirit of effective field theory, the correct counting rule can be obtained by integrating out the hard scale, modulo possible additional factors due to helicity suppression and so on. The counting rule is demonstrated using the  $e^+e^- \rightarrow VP$  process. We discussed productions of the  $Z_c^\pm(\bar{c}c\bar{u}d/\bar{c}c\bar{d}u)$ ,  $X(3872)$  and others in a few hard exclusive processes.

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